The Traveling Wave IMPATT Mode: Part II—The Effective Wave Impedance and Equivalent Transmission Line

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Abstract—**The coupling between a microstrip and a distributed IMPATT diode was investigated in a field analysis. An effective wave impedance in the traveling wave diode can be defined as the ratio of the space-average transverse electric and magnetic fields. This impedance is related to an effective characteristic impedance by a geometry factor. Thus the coupling question is reduced to the coupling between two transmission lines.**

In addition the diode is modeled in an equivalent transmission line. The equivalent series impedance and shunt admittance are found. The shunt admittance is approximately equal to the admittance (per unit length) of a discrete diode of identical doping profile.

The coupling analysis presented here seems applicable to microstrip interfaces to traveling wave structures other than the IMPATT diode.

I. Introduction

THE EXISTENCE of a small-signal traveling wave IMPATT mode was established in a publication by the same authors [1] and the following is based on the previous findings. The same nomenclature is used as well as the same numbering system of the equations and figures.

Of particular interest to the user of an IMPATT traveling wave amplifier (TWA) is the interface to the external microwave circuit. As will be shown, the question of coupling into and out of an IMPATT traveling wave structure requires new definitions of the wave and characteristic impedances. The situation is illustrated in Fig. 8[1] and is more instructive than realistic because in a practical case the microstrip will be much larger than the device. The transition from a parallel plate waveguide (in reality a microstrip) with a dielectric thickness w to the IMPATT TWA is shown.

In previous works [2], [3] the question of coupling to the TWA was treated as a transmission-line matching problem, i.e., after having established a characteristic impedance in the TWA as a voltage-to-current ratio of an incident wave, a match is achieved by adjusting the external circuit impedance accordingly.

It seems, however, that the question of coupling to the distributed diode deserves slightly more attention. The statement that a transmission line is match-terminated superficially means that the load impedance accommodates exactly the voltage-to-current ratio of the incoming wave and no reflected wave is needed to satisfy the load boundary condition. This statement, however, is only a summary of the fundamental requirement that the ratio of incoming transverse electric and magnetic *fields* is accommodated by the load surface impedance at every point. In practice this condition is difficult to meet. However, for the matched load case, the reflections due to pointwise mismatches of the wave impedance and the surface impedance result in higher order modes which normally do not propagate.

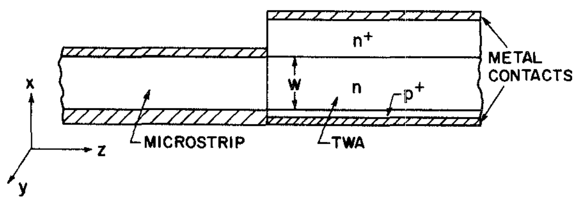

Fig. 8. Transition from microstrip to IMPATT TWA.

Applied to the problem at hand, the question of coupling to the IMPATT mode with its irregular traveling wave in the transverse direction should first be treated as a field problem and later, if possible, be translated into a transmission-line model.

II. The Effective Wave Impedance

The problem is attacked by expressing the electromagnetic fields in the microstrip in the appropriate modal expansion (modeling the microstrip as a parallel plate transmission line) and then matching the tangential electromagnetic fields at the interface. As an expansion the TE to y modes were chosen since no y-directed electric field exists in the TWA:[2]

$$E_x = E_n^{\pm} \cos\left(\frac{n\pi}{w}x\right)e^{\gamma_n^{\pm} z} \qquad H_x = 0$$

$$E_y = 0 \qquad H_y = \frac{\omega\epsilon_M}{j\gamma_n^{\pm}} E_n^{\pm} \cos\left(\frac{n\pi}{w}x\right)e^{\gamma_n^{\pm} z}$$

$$E_z = \frac{n\pi}{w\gamma_n^{\pm}} E_n \sin\left(\frac{n\pi}{w}x\right)e^{\gamma_n^{\pm} z} \qquad H_z = 0$$

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[1]*Editor's Note*: The enumerations of figures and equations in this paper are a continuation from [1].

[2]See footnote 1.

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with

$$\left(\frac{n\pi}{w}\right)^2 + \gamma_n^{\pm 2} = \omega^2 \epsilon_M \mu, \qquad n = 0, 1, 2, \cdots. \tag{40}$$

$E_n^{\pm}$ and $\gamma_n^{\pm}$ are the amplitude and propagation constant of the nth mode traveling in the positive (+) or negative (−) z directions. ϵ_M is the permittivity of the microstrip dielectric. All other symbols have their standard meaning.

For a microstrip with $\epsilon_{M_r} = 3$ and whose dielectric thickness equals a typical depletion layer thickness, $w = 7$ µm as shown in Fig. 8, the cutoff frequency of the first higher mode ($n=1$) is $f_c = 1/(2w\sqrt{\epsilon_M \mu}) = 12.4 \times 10^{12}$ Hz and hence only the fundamental TEM mode propagates.

Setting $z=0$ at the microstrip–TWA interface and assuming an incident wave in the microstrip causing only an incident wave in the TWA (in other words reflections in the TWA are excluded) we can equate the electric and magnetic fields on both sides.

$$E_0^+ + \sum_{n=0}^{\infty} E_n^- \cos\left(\frac{n\pi}{w}x\right) = \mathring{E}_x(x), \qquad 0 \leqslant x \leqslant w \tag{41}$$

$$\frac{\omega\epsilon_M}{j}\left[\frac{E_0^+}{\gamma_n^+} + \sum_{n=0}^{\infty} \frac{E_n^-}{\gamma_n^-} \cos\left(\frac{n\pi}{w}x\right)\right] = \mathring{H}_y(x), \qquad 0 \leqslant x \leqslant w \tag{42}$$

where $\mathring{E}_x(x)$ and $\mathring{H}_y(x)$ are the respective electric and magnetic fields in the TWA at the interface. Integrating (41) and (42) over the interface $0 \leqslant x \leqslant w$ and applying orthogonality concepts results in the following relations of the dominant mode amplitudes:

$$(E_0^+ + E_0^-) = \frac{1}{w}\int_0^w \mathring{E}_x(x)\,dx \equiv \overline{E}_x \tag{43}$$

$$\frac{1}{\eta_M}(E_0^+ - E_0^-) = \frac{1}{w}\int_0^w \mathring{H}_y(x)\,dx \equiv \overline{H}_y. \tag{44}$$

Here the intrinsic wave impedance of the microstrip was introduced as $\eta_M = \sqrt{\mu/\epsilon_M}$.

Hence it is the *average* electric and magnetic field which couples to the dominant mode in the microstrip. Introducing the reflection coefficient

$$\Gamma = E_0^- / E_0^+ \tag{45}$$

it is found that

$$\Gamma = \frac{\hat{\eta} - \eta_M}{\hat{\eta} + \eta_M} \tag{46}$$

where $\hat{\eta}$ defines an effective wave impedance of the TWA which is seen by the microstrip as the load surface impedance:

$$\hat{\eta} \equiv \frac{\int_0^w \mathring{E}_x(x)\,dx}{\int_0^w \mathring{H}_y(x)\,dx} = \frac{\overline{E}_x}{\overline{H}_y}. \tag{47}$$

The effective wave impedance is the ratio of the space-average transverse electric and space-average transverse magnetic fields. By multiplying the intrinsic wave impedance of the microstrip η_M and the effective wave impedance of the TWA $\hat{\eta}$ by w/d, d being the width of the microstrip and the TWA, respective expressions for the characteristic impedances are obtained:

$$Z_{0M} \simeq \eta_M (w/d) \tag{48}$$

$$\hat{Z}_0 \simeq \hat{\eta}(w/d) \tag{49}$$

and the reflection coefficient Γ can be rewritten

$$\Gamma = \frac{\hat{Z}_0 - Z_{0M}}{\hat{Z}_0 + Z_{0M}}. \tag{50}$$

It is clear from (47) and (49) that the effective characteristic impedance is not simply the voltage over current ratio as suggested in [3] but rather the ratio of the voltage over that current component which is caused by the average tangential magnetic field.

The method described so far is general and applies to other transitions from a microstrip to a traveling wave structure. For the IMPATT traveling wave mode the calculations required by (47) were performed.

The transverse electric field was given in [1, eq.(32)]. Maxwell's first equation provides the magnetic ac field in the diode:

$$\vec{\tilde{H}} = \frac{j}{\omega\mu}\nabla \times \vec{\tilde{E}} = \frac{j}{\omega\mu}\left(\frac{\partial \tilde{E}_x}{\partial z} - \frac{\partial \tilde{E}_z}{\partial x}\right)\hat{a}_y. \tag{51}$$

$\hat{a}_y$ is the unit vector in the y direction. Using (17), (19), and (32) yields the ac electron density as

$$\tilde{n} = \mathring{n} e^{\gamma z} = -j\frac{\epsilon\omega}{q v_s}\left[a_n A - a_n B - \frac{gf}{a_n^2 - 1}\mathring{E}_x(0)\right] e^{-jx_n} e^{\gamma z}. \tag{52}$$

Substituting this result into (39) and integrating with respect to z gives the longitudinal field E_z. This along with (32) in (51) yields the expression for the transverse magnetic field.

$$\mathring{H}_y(x_n) = -\frac{j\omega\epsilon}{\gamma}\left[A e^{j a_n x_n} + B e^{-j a_n x_n} + \frac{\gamma^2 g \mathring{E}_x(0)}{\left(\frac{\omega}{v_s}\right)^2 (a_n^2 - 1)} e^{-jx_n}\right] \tag{53}$$

with γ being the propagation constant of the IMPATT mode. Integration of (32) and (53) as required by (47) and substituting for A and B in terms of $e_x(0)$ from (33) yields the effective wave impedance

$$\hat{\eta} = -j\omega\mu/\gamma. \tag{54}$$

It is noted that this impedance reduces to the intrinsic wave impedance $\eta = \sqrt{\mu/\epsilon}$ for the case of no bias current $J_{dc} = 0$ in which case $g = 0$ and $a_n = 0$ resulting, according to (28), in the propagation constant $\gamma = -j\omega\sqrt{\mu\epsilon}$: the injection-free depletion layer essentially acts as a parallel plate waveguide. Fig. 9 is a plot of the real and imaginary (dashed) part of $\hat{\eta}$ versus frequency

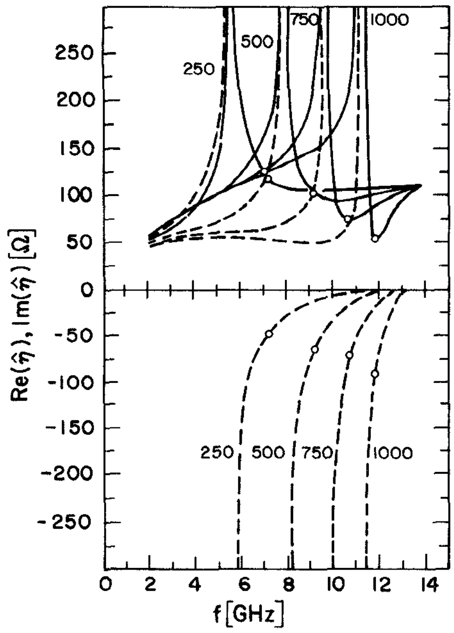

Fig. 9. Real and imaginary (dashed) part of the effective wave impedance versus frequency with current density as curve parameter. Circles mark points of maximum gain.

for various bias current densities.[3] At the cutoff frequency

$$f_c = \frac{1}{\pi}\sqrt{\frac{(m+1)}{2\tau_1\epsilon(-E_c)}}\cdot\sqrt{J_{dc}} \tag{55}$$

the propagation constant γ goes to zero (compare to [1, fig. 4]) and consequently $\hat{\eta}$ has a pole at this frequency. The imaginary part switches sign from inductive to capacitive. At cutoff, the diode does not accept any power. Above f_c amplification occurs, and the points of maximum gain are marked. It is seen that the real part at points of maximum gain decreases with increasing current density while the capacitive imaginary part increases. The impedance magnitude, however, is approximately constant at these points and of the order of the intrinsic wave impedance $\eta = \sqrt{\mu/\epsilon} \simeq 110\ \Omega$. An upper frequency bound for active operation is suggested by Fig. 6 in [1] and can be found from (28) and (34) by requiring that $\gamma = j\beta$ be purely imaginary:

$$f_u = v_s/w. \tag{56}$$

Thus the active frequency band has the cutoff frequency as it is lower and two times the transit time frequency as its upper bound. The similarity with the discrete diode operation is evident. At the upper frequency limit, the equivalent wave impedance approaches the intrinsic impedance and is purely real.

It was pointed out earlier that the transition in Fig. 8 is not very realistic in that practical dielectric thicknesses of microstrips are considerably larger than the depletion layer width. For typical dimensions of the diode ($w = 7$ µm, $d = 0.15$ mm) the equivalent characteristic impedance magnitude at points of maximum gain becomes approximately $|\hat{Z}_0| \simeq 110 \times 7 \times 10^{-4} \times 0.015\ \Omega \simeq 5.1\ \Omega$. To match standard 50-Ω microstrip circuitry to this impedance level a quarter-wave step transformer was considered and will be treated in a later publication. The dielectric thickness is stepped up and the first step occurs at the diode terminal guaranteeing a reasonable dielectric thickness.

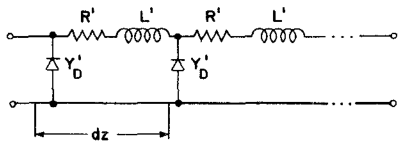

Fig. 10. Transmission-line model of IMPATT TWA. All primed parameters are distributed and taken per unit length.

III. Equivalent Transmission Line

In the following the small-signal parameters of an equivalent transmission line are found, i.e., the complex distributed shunt admittance as well as the series impedance of the transmission line seen in Fig. 10. The method is to equate the complex propagation constant of the TWA as well as its effective characteristic impedance with the respective quantities of the transmission line:

$$\gamma = \sqrt{Z'Y'} \tag{57}$$

$$\hat{Z}_0 = -\frac{j\omega\mu}{\gamma}\frac{w}{d} = \sqrt{\frac{Z'}{Y'}}\ . \tag{58}$$

Here Z' and Y' are the series impedance and shunt admittance per unit length of the transmission line. By eliminating Y' from (57) and (58) the series impedance is found to be independent of γ and purely imaginary

$$Z' = j\omega(\mu w/d). \tag{59}$$

This is not surprising since the conductors were modeled as perfect. Hence, the series impedance consists only of the inductance

$$L' = \mu w/d \tag{60}$$

of a parallel plate transmission line. The shunt admittance becomes with (57) and (58)

$$Y' = j(\gamma^2 d/\omega\mu w). \tag{61}$$

In order to evaluate this expression explicitly, an expression for the propagation constant γ would be needed. The propagation constant γ, however, is given in terms of the normalized a_n (28), which in turn are found from the transcendental characteristic equation (34). From (28) it can be argued that the auxiliary constant a_n^2 must be small in magnitude since it is the sum of $c_n^2 = \epsilon\mu v_s^2$ (typically 1.3×10^{-6}) and the normalized propagation constant squared, $\gamma_n^2 = (\gamma v_s/\omega)^2$ with $\omega|v_s$ being typically $2\pi \times 10^3$ cm^{-1} which is much larger than the real or imaginary part of γ.

[3]Fig. 9 is based on the same diode specifications as the example in [1]: Material: Si; $\epsilon_r = 11.8$; donor doping in depletion region $N_D = 2.2 \times 10^{15}$ cm^{-3}; depletion region width $w = 7$ µm; scattering limited velocity $v_s = 10^7$ cm/s; critical field $E_c = -2.66 \times 10^5$ V/cm; breakdown voltage $V_B = 104$ V.

Hence, it is justified in a first-order approximation to neglect a_n^2 with respect to 1 and to expand the sine and cosine functions in the characteristic equation (34) resulting in the explicit expression for a_n

$$a_n^2 \simeq \frac{c^2}{1-\dfrac{jw_n(1+1/g)}{e^{-jw_n}-1}}. \tag{62}$$

The constant $f=1-c^2$ was approximated by 1. The propagation constant γ is then found from (28)

$$\gamma^2 = \left(\frac{\omega}{v_s}\right)^2 (a_n^2 - c^2) \simeq \left(\frac{\omega}{v_s}\right)^2 \frac{c^2}{j\dfrac{e^{-jw_n}-1}{w_n(1+1/g)}-1}. \tag{63}$$

The inverse of the shunt admittance can now be written as

$$Z_s' = \frac{1}{Y'} = \frac{w^2}{2d\epsilon v_s}\left[\frac{1-\cos(w_n)}{\left(1-\dfrac{\omega^2}{\omega_c^2}\right)\dfrac{w_n^2}{2}}\right] + \frac{j}{C_d'\omega}\left[\frac{\sin(w_n)}{w_n} - 1 - \frac{\sin(w_n)}{w_n\left(1-\dfrac{\omega_c^2}{\omega^2}\right)}\right]. \tag{64}$$

Here the angular cutoff frequency $\omega_c = 2\pi f_c$ was introduced from (55) as well as the distributed depletion layer capacitance $C_d' = \epsilon d/w$.

The interesting aspect of this result is that the shunt impedance is exactly equal to the complex small signal impedance of the lumped diode as presented in [4] for the case of zero-avalanche region width and no parasitic resistance (since the TWA was modeled between perfect conductors). Since the TWA was conceived as an ideal READ diode with the carrier generation occurring in an infinitesimal avalanche zone (it was a boundary condition in the analysis) this result is not surprising. The fact, however, that the shunt admittance of the transmission line can be thought of with satisfying accuracy[4] as the distributed admittance of the lumped diode is significant since it ultimately justifies the approach chosen in [2] to model the distributed diode in terms of distributed transmission-line parameters. It also suggests that a large-signal model could be based on transmission-line concepts with the shunt admittance being a function of the ac voltage amplitude.

[4]When the numerical results for γ from (34), (38) were compared with the equivalent ones from (63), a deviation of less than 10 percent was noticed in real and imaginary part.

IV. Conclusions

The approximate electromagnetic-field analysis of the coupling problem between a microstrip and a traveling wave structure indicates that the effective wave impedance and the closely related effective characteristic impedance are the significant quantities. In contrast to previous definitions the effective wave impedance is the ratio of the space-average transverse electric and magnetic fields. It is inversely proportional to the propagation constant and inductive in the attenuating and capacitive in the active regime. The concept of an effective characteristic impedance facilitates a simple method of coupling between an external microstrip and the diode.

By modeling the distributed diode in an equivalent transmission line the coupling question is reduced to a standard transmission line problem. It is significant to notice that the equivalent shunt admittance is approximately equal to the admittance (per unit length) of a discrete diode. This would suggest that further analysis (in particular large signal analysis) can be carried out on the much simpler transmission-line model rather than on the field model.

References

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